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Nonlinear Dynamic Analysis of Cracked Beam on Elastic Foundation Subjected to Moving Mass

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Abstract**—*This paper presents a finite element algorithm for nonlinear dynamic analysis of cracked beams on an elastic foundation subjected to moving mass. Quantity surveying with parameters of varied cracks, foundation and loads shows their influence levels on the nonlinear dynamic response of the beams. The findings of the paper are the basis for the analysis, evaluation, and diagnosis of damages of beam structures on the elastic foundation subjected to moving loads, in which the common defects of the beams such as cracks are considered in order to improve the system's operational efficiency in a wide range of engineering applications.***

Keywords—Nonlinear, cracked beam, elastic foundation, moving masses.

I. INTRODUCTION

Beams on the foundations are usually modeled to calculate the structures of railway works and civil engineering. During the use, there are many different causes that can cause weakened defects for beams, one of which is cracks. The appearance of cracks will reduce bearing capacity of the beams, which leads to the risk of damage to the building. Salih N Akour [1] analyzed the nonlinear dynamics of beams on the elastic foundation subjected to evenly distributed moving force by analytical methods. Also using analytical methods, Oni and Awodola [2], Tiwari and Kuppa [3] analyzed the dynamics of Bernoulli - Euler beams on the elastic foundation subjected to moving masses. Haitao Yu and Yong Yuan [4] have focused on the analytical solution of an infinite Euler-Bernoulli beam on a viscoelastic foundation subjected to arbitrary dynamic loads. Şeref Doğuşcan AKBAŞ [5] investigated the free Vibration and Bending of Functionally Graded Beams on Winkler's elastic foundation using Navier method. Nguyen Dinh Kien, Tran Thi Thom [6] studied the influences of dynamic moving forces on the Functionally Graded Porous-Nonuniform beams. D. Froio1, R. Moioli1, E. Rizzi [7] and D. T. Pham, P. H. Hoang and T. P. Nguyen [8] used the nonlinear elastic foundation and New Non-Uniform Dynamic Foundation applied to analyzed response of beam subjected to moving load and the results show that the influence of velocity has effects significantly on dynamic response of structures. N. T. Khiem, P. T. Hang [9] used a spectral method applied to analyzed response of a multiple Cracked Beam subjected to moving load.

Using analytical and finite element methods, Murat. R and Yasar. P [10], Mihir Kumar Sutar [11], Animesh C. and Tanuja S. V [12], Shakti P Jena, Dayal R Parhi, P C Jena [13], A.C.Neves, F.M.F. Simoes, A.Pinto da Costa [14], Hui Ma et al. [15] analyzed the dynamics of cracked beams subjected to moving mass.

Arash Khassetarash, Reza Hassannejad [16] investigated the Energy dissipation caused by fatigue crack in beam-like cracked structures. Erasmo Viola, Alessandro Marzani, Nicholas Fantuzzi [17] used finite element method applied to studied effect of cracks on flutter and divergence instabilities of cracked beams under subtangential forces.

M Attar et al. [18] analyzed the dynamics of cracked beams on the elastic foundation subjected to moving harmonic loads by analytic method, on the basis of using Timoshenko beam model.

So far, there are various researches of beams on elastic foundation under transfer (mass, force, oscillation system). However, for cracked beam on the elastic foundation under moving loads(or masses), most methods reply on analytical approaches which are really applied to simple loading conditions. In this paper, we develop a numerical approach based on finite element method for analyzing the dynamics of beams on elastic foundation under moving masses. We investigate the influence of the elastic foundation, load speeds and location cracks in the dynamic response of the beams. Note that finding analytical solutions of such beam problems under arbitrary loading conditions are really challenging and no research is sufficiently carried out yet. Such a problem will be addressed in this paper.

II. FINITE ELEMENT FORMULATION AND THE GOVERNING EQUATIONS

A damaged beam has an open crack located at the mid-section of the beam at position x = x0. The beam on an elastic foundation described by an elastic spring system to one direction perpendicular to the axis of the beam, which has the stiffness kt subjected to traversing mass 'm' at speed 'v' as in Fig. 1. The dimensions of the damaged

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beam are as follows, width = b, thickness = h, length = L, crack depth = d.

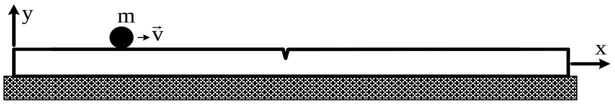

Fig. 1. Cracked beam on the foundation subjected to moving mass

For finite element model formulation the following assumptions are made: Elastic isotropic materials, cracks do not develop, and mass 'm' is always in contact the surface of the beam.

The Timoshenko theory describes the displacement field components (u,v,w) of an arbitrary point (x,y,z) on the beam cross-section can be expressed as

$$\begin{cases} u = u_0(x,t) - y\varphi_z(x,t), \\ v = v_0(x,t), \\ w = 0, \end{cases} \tag{1}$$

where u0, v0 are respectively the x and y components of the total displacement vector of the point (x,0,0) on the beam neutral axis at time t, and φz is the cross-section rotation about the z-axis. The subscript "0" represents axis x (y = 0, z = 0; x contains the cross section centroids of the undeformed beam, that will be often designed as middle line or reference line, in bending it coincides with neutral line). The x-coordinate is defined along the beam length, y-coordinate is along the height and the z-coordinate is along the width. The strain-displacement relations are as

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 = \dfrac{\partial u_0}{\partial x} - y\dfrac{\partial \varphi_z}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v_0}{\partial x}\right)^2, \\ \gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = \dfrac{\partial v_0}{\partial x} - \varphi_z, \end{cases} \tag{2}$$

$$\{\varepsilon\} = \left\{\varepsilon^L\right\} + \left\{\varepsilon^{NL}\right\} = \begin{Bmatrix} \varepsilon_x^L \\ \gamma_{xy}^L \end{Bmatrix} + \begin{Bmatrix} \varepsilon_x^N \\ \gamma_{xy}^N \end{Bmatrix}, \tag{3}$$

where $\left\{\varepsilon^L\right\}$ is the linear part of the strain and $\left\{\varepsilon^L\right\}$ is the nonlinear part given by:

$$\varepsilon_x^L = \frac{\partial u_0}{\partial x} - y\frac{\partial \varphi_z}{\partial x},\ \gamma_{xy}^L = \frac{\partial v_0}{\partial x} - \varphi_z,$$
$$\varepsilon_x^{NL} = \frac{1}{2}\left(\frac{\partial v_0}{\partial x}\right)^2,\ \gamma_{xy}^{NL} = 0. \tag{4}$$

The stresses are related to the strain by Hooke's law:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} E & 0 \\ 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \gamma_{xy} \end{Bmatrix} = [D]\{\varepsilon\} = [D]\left\{\varepsilon^L\right\} + [D]\left\{\varepsilon^{NL}\right\}, \tag{5}$$

where E is the Young's modulus of the material, G is the shear modulus and [D] is the material matrix.

2.1. Equation of motion of beam element with out crack

The equation of motion is derived by the principle of virtual work [19], [21]:

$$\delta W_V + \delta W_{in} + \delta W_E = 0, \tag{6}$$

where δW_V is the virtual work of internal forces, δW_{in} is the virtual work of inertia forces and δW_E is the vertual work of external forces due to a virtual displacement. They are defined as:

$$\begin{aligned} \delta W_V &= -\int_V \delta\{\varepsilon\}^T\{\sigma\} = -\int_V \delta\left\{\varepsilon^L\right\}^T [D]\left\{\varepsilon^L\right\} dV - \\ &-\int_V \delta\left\{\varepsilon^L\right\}^T [D]\left\{\varepsilon^{NL}\right\} dV - \int_V \delta\left\{\varepsilon^{NL}\right\}^T [D]\left\{\varepsilon^L\right\} dV \\ &-\int_V \delta\left\{\varepsilon^{NL}\right\}^T [D]\left\{\varepsilon^{NL}\right\} dV = \\ &= -\{\delta q\}^T [K_1]\left\{q^e\right\} - \{\delta q\}^T\left[K_2\left(\left\{q^e\right\}\right)\right]\left\{q^e\right\} - \\ &-\{\delta q\}^T\left[K_3(\{q\})\right]\left\{q^e\right\} - \{\delta q\}^T\left[K_4\left(\left\{q^e\right\}\right)\right]\left\{q^e\right\}, \end{aligned} \tag{7}$$

In this equation [K1] is a matrix of constant terms, [K2({q})] and [K3({q})] are matrices that depend linearly on the generalized displacements and [K4({q})] is a matrix that depends quadratically on the generalized displacements, {qe} is the displacements vector. The linear stiffness matrix [K1], nonlinear stiffness matrices [K2], [K3] and [K4] have the following form:

$$[K_1] = \begin{bmatrix} [K_1]_{11} & [0] & [0] \\ [0] & [K_1]_{22} & [K_1]_{23} \\ [0] & [K_1]_{23}^T & [K_1]_{33} \end{bmatrix},\ [K_2] = \begin{bmatrix} [K_2]_{11} & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{bmatrix},$$
$$[K_4] = \begin{bmatrix} [K_4]_{11} & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{bmatrix},\ [K_3] = 2[K_2], \tag{8}$$

where

$$[K_1]_{11} = E\int_V \frac{d\left[N^u\right]^T}{dx}\frac{d\left[N^u\right]}{dx} dV,$$
$$[K_1]_{22} = \lambda G\int_V \frac{d\left[N^v\right]^T}{dx}\frac{d\left[N^v\right]}{dx} dV, \tag{9}$$

$$[K_1]_{23} = -\lambda G\int_V \frac{d\left[N^v\right]^T}{dx}\left[N^{\varphi_z}\right] dV,\ [K_1]_{33}$$
$$= \int_V \left(y^2 E\frac{d\left[N^{\varphi_z}\right]^T}{dx}\frac{d\left[N^{\varphi_z}\right]}{dx} + \lambda G\left[N^{\varphi_z}\right]^T\left[N^{\varphi_z}\right] \right) dV, \tag{10}$$

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$$[K_2]_{11} = \frac{1}{2} E \int_V \frac{d\left[N^u\right]^T}{dx} \frac{d\left[N^v\right]}{dx} \frac{\partial v_0}{\partial x} dV,$$

$$[K_4]_{11} = \frac{1}{2} E \int_V \frac{d\left[N^v\right]^T}{dx} \frac{d\left[N^v\right]}{dx} \left(\frac{\partial v_0}{\partial x}\right)^2 dV. \quad (11)$$

$$\delta W_{in} = -\int_V \rho \{\delta d\}^T \{\ddot{d}\} = -\{\delta d\}^T \left[M^e\right]\left\{\ddot{q}^e\right\}, \quad (12)$$

$$\delta W_E = \int_V \{\delta d_0\}^T \{F_0\} dV = \{\delta d\}^T \left\{F^e\right\}, \quad (13)$$

where $[N^u]$, $[N^v]$ and $[N^{\varphi z}]$ are the row vectors of longitudinal, transverse along y and rotational about z shape functions, respectively, $\left\{\ddot{q}^e\right\}$ is the acceleration vector, ρ is the density of the beam, $[M^e]$ is the mass matrix, the vector of virtual displacement components will be represented by $\{\delta d\}$ and can be written as $\{\delta d\} = \{\delta u \;\; \delta v \;\; 0\}^T$ and $\{\delta d_0\}$ is the vertual displacements on the middle line, $\{F_0\}$ is the external forces on the middle line, $\{F^e\}$ is the generalized external forces.

The mass matrix [M] and vector of generalized external forces {F} have following form:

$$\left[M^e\right] = \begin{bmatrix} [M]_{11} & [0] & [0] \\ [0] & [M]_{22} & [0] \\ [0] & [0] & [M]_{33} \end{bmatrix}, \quad (14)$$

where

$$[M]_{11} = \rho \int_V \left[N^u\right]^T \left[N^u\right] dV, [M]_{22} = \rho \int_V \left[N^v\right]^T \left[N^v\right] dV, [M]_{33} = \rho \int_V y^2 \left[N^{\varphi_z}\right]^T \left[N^{\varphi_z}\right] dV. \quad (15)$$

$$\{F_0\} = \{F_x \;\; F_x \;\; M_z\}^T, \left\{F^e\right\} = \left\{F_{u_0} \;\; F_{v_0} \;\; F_{\varphi_z}\right\}^T, \quad (16)$$

where

$$F_{u_0} = \int_V \left[N^u\right] F_x dV, \; F_{v_0} = \int_V \left[N^v\right] F_y dV, \quad (17)$$

$$F_{\varphi_z} = \int_V \left[N^{\varphi_z}\right] M_z dV.$$

Substituting equations (7), (12) and (13) into (6), the following nonlinear equation of motion of the beam without crack is obtained

$$\left[M^e\right]\left\{\ddot{q}^e\right\} + \left[K^e\left(\left\{q^e\right\}\right)\right]\left\{q^e\right\} = \left\{F^e\right\}, \quad (18)$$

In case with the damping force $\left\{f_d^e\right\} = \left[C^e\right]\left\{\dot{q}^e\right\}$, equation (18) becomes

$$\left[M^e\right]\left\{\ddot{q}^e\right\} + \left[C^e\right]\left\{\dot{q}^e\right\} + \left[K^e\left(\left\{q^e\right\}\right)\right]\left\{q^e\right\} = \left\{F^e\right\}, \quad (19)$$

where

$$\left[K^e\left(\left\{q^e\right\}\right)\right] = [K_1] + \left[K_2\left(\left\{q^e\right\}\right)\right] + \left[K_3\left(\left\{q^e\right\}\right)\right] + \left[K_4\left(\left\{q^e\right\}\right)\right],$$

$\left[C^e\right]$ is the damping matrix of element, $\left\{\dot{q}^e\right\}$ is velocity vector.

2.2. Beam element with crack

Considering the beam element with crack, stiffness matrix of the element $\left[K_c^e\right]$ is identified as [11], [20]

$$\left[K_c^e\left(\left\{q^e\right\}\right)\right] = \left[K^e\left(\left\{q^e\right\}\right)\right] - [K_c], \quad (20)$$

where $[K_c]$ is the stiffness matrix of weak beam element due to cracks, and the above matrices can be formulated as:

$$[K_c] = \begin{bmatrix} k_{11} & k_{12} & -k_{11} & k_{14} \\ k_{12} & k_{22} & -k_{12} & k_{24} \\ -k_{11} & -k_{12} & k_{11} & -k_{14} \\ k_{14} & k_{24} & -k_{14} & k_{44} \end{bmatrix},$$

$$\left[K^e\right] = \frac{EJ_0}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}, \quad (21)$$

with components in equation (21)

$$k_{11} = \frac{12E(J_0 - J_c)}{l_e^4}\left[\frac{2l_c^2}{l_e^2} + 3l_c\left(\frac{2\eta}{l_e^2} - 1\right)^2\right],$$

$$k_{12} = \frac{12E(J_0 - J_c)}{l_e^3}\left[\frac{l_c^3}{l_e^2} + l_c\left(2 - \frac{7\eta}{l_e} + \frac{6\eta^2}{l_e^2}\right)\right],$$

$$k_{14} = \frac{12E(J_0 - J_c)}{l_e^3}\left[\frac{l_c^3}{l_e^2} + l_c\left(2 - \frac{5\eta}{l_e} + \frac{6\eta^2}{l_e^2}\right)\right],$$

$$k_{22} = \frac{12E(J_0 - J_c)}{l_e^2}\left[\frac{3l_c^2}{l_e^2} + 2l_c\left(\frac{3\eta}{l_e} - 2\right)^2\right],$$

$$k_{24} = \frac{12E(J_0 - J_c)}{l_e^2}\left[\frac{3l_c^2}{l_e^2} + 2l_c\left(2 - \frac{9\eta}{l_e} + \frac{9\eta^2}{l_e^2}\right)^2\right],$$

$$k_{44} = \frac{12E(J_0 - J_c)}{l_e^2}\left[\frac{3l_c^2}{l_e^2} + 2l_c\left(\frac{3\eta}{l_e} - 1\right)^2\right]$$

where J_0, J_c , respectively, are the moments of inertia of the beam cross section for Oz axis at non-cracked positions and at the positions of cracks; $l_c = 1,5d$ (*d* is the depth of the crack), l_e is the length of the element, *E* is the modulus of elasticity, η is the distance from the left end of beam element to the crack.

Considering that the lost mass due to cracks is little compared to the overall mass of the element.

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2.3. Beam element on elastic foundation

Stiffness matrix of the beam element on elastic foundation $\left[K_{bf}^{e}\right]$ is identified by [3]:

$$\left[K_{bf}^{e}\left(\left\{q^{e}\right\}\right)\right]=\left[K^{e}\left(\left\{q^{e}\right\}\right)\right]+\left[K_{f}^{e}\right], \quad (22)$$

where $\left[K^{e}\left(\left\{q^{e}\right\}\right)\right]=\left[K_{c}^{e}\left(\left\{q^{e}\right\}\right)\right]$ correlates with the cracked beam element, $\left[K_{f}^{e}\right]$ is the stiffness matrix related to an elastic foundation.

2.4. Nodal load vector element beam on elastic foundation under moving mass

According to FEM method, when a moving load is involved in the working of the system, due to the position change property of the load over time, so at each point of time, the moving load acts on one beam element. Considering the beam element on elastic foundation subjected to the moving mass m, the force P(t) acts on m (Fig. 3).

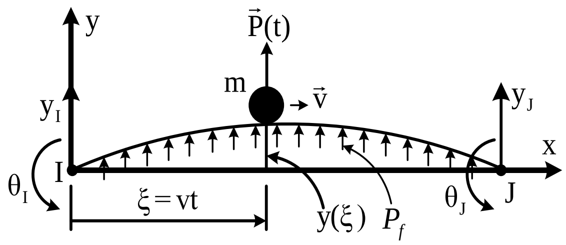

Fig. 3. Beam element on elastic foundation under moving mass.

The force of the moving mass acting on the beam at the coordinate x = ξ = vt is:

$$R(x,t)=P(t)-m\left.\frac{d^{2}y(x,t)}{dt^{2}}\right|_{x=\xi}, \quad (23)$$

where y(x,t) is element deflection, $\frac{d^2y}{dt^2}$ is the absolute acceleration in the y direction.

After taking the derivative of the deflection function, the expression (23) is rewritten as

$$R(x,t)=P(t)-m\left.\left(\frac{\partial^{2}y}{\partial t^{2}}+2\frac{\partial^{2}y}{\partial x\partial t}v+\frac{\partial^{2}y}{\partial x^{2}}v^{2}\right)\right|_{x=\xi}. \quad (24)$$

The acting force (24) is described by the distributed force p(x,t) as:

$$p(x,t)=R(x,t)\cdot\delta(x-vt). \quad (25)$$

In case, beam on elastic foundation:

$$p(x,t)=R(x,t)\cdot\delta(x-vt)-k_{0}y(x), \quad (26)$$

where δ(x) is denotes the Dirac-Delta function, k0 – foundation modulus.

Therefore, the force vector is:

$$\left\{F^{e}\right\}=\int_{0}^{l_e}[N]^{T}p(x,t)dx= \int_{0}^{l_e}\delta(x-\xi)[N]^{T}R(x,t)dx-\int_{0}^{l_e}k_{0}[N]^{T}y(x)dx, \quad (27)$$

where, $[N]=\begin{bmatrix}\left[N^{u}\right] & [0] & [0]\\ [0] & \left[N^{v}\right] & [0]\\ [0] & [0] & \left[N^{\varphi_z}\right]\end{bmatrix}$ is the matrix of shape functions of flexural beam element.

Substituting $y(x)=[N]\left\{q^{e}\right\}$ into Eq. (24) we get

$$R(x,t)=P(t)-m\left.\left([N]\left\{\ddot{q}^{e}\right\}+2v[N']\left\{\dot{q}^{e}\right\}+[N'']\left\{q^{e}\right\}v^{2}\right)\right|_{x=\xi}, \quad (28)$$

where $[N']=\frac{\partial N}{\partial x}, [N'']=\frac{\partial^{2}N}{\partial x^{2}}$.

Substituting equations (28) into (26) and paying attention to the nature of the Delta-Dirac function, equation (27) may be rewritten as

$$\left\{F^{e}(t)\right\}=\left\{P^{e}(t)\right\}-\left[M_{P}^{e}\right]\left\{\ddot{q}^{e}\right\}-\left[C_{P}^{e}\right]\left\{\dot{q}^{e}\right\} -\left[K_{P}^{e}\right]\left\{q^{e}\right\}-\left[K_{f}^{e}\right]\left\{q^{e}\right\}, \quad (29)$$

where

$$\left\{P^{e}(t)\right\}=\int_{0}^{l_e}[N]^{T}\delta(x-\xi)P(t)dx=\left[N(\xi)\right]^{T}P(t), \quad (30)$$

$$\left[M_{P}^{e}\right]=m\int_{0}^{l_e}\delta(x-\xi)[N]^{T}[N]dx=m\left[N(\xi)\right]^{T}\left[N(\xi)\right] \quad (31)$$

$$\left[C_{P}^{e}\right]=2vm\int_{0}^{l_e}\delta(x-\xi)[N]^{T}[N']dx=2vm\left[N(\xi)\right]^{T}\left[N'(\xi)\right], \quad (32)$$

$$\left[K_{P}^{e}\right]=v^{2}m\int_{0}^{l_e}\delta(x-\xi)[N]^{T}[N'']dx=mv^{2}\left[N(\xi)\right]^{T}\left[N''(\xi)\right], \quad (33)$$

$$\left[K_{f}^{e}\right]=\int_{0}^{l_e}k_{0}\left[N(x)\right]^{T}\left[N(x)\right]dx. \quad (34)$$

Substituting equation (29) into equation (19), we get the equations of motion governing the nonlinear dynamic response of the beam element on elastic foundation subjected to moving mass

$$\left(\left[M^{e}\right]+\left[M_{P}^{e}\right]\right)\left\{\ddot{q}^{e}\right\}+\left(\left[C^{e}\right]+\left[C_{P}^{e}\right]\right)\left\{\dot{q}^{e}\right\} +\left(\left[K_{bf}^{e}\left(\left\{q^{e}\right\}\right)\right]+\left[K_{P}^{e}\right]\right)\left\{q^{e}\right\}=\left\{P^{e}(t)\right\}, \quad (35)$$

2.5. Governing differential equations for total system

Assembling all elements matrices and nodal force vectors, the governing equations of motions of the cracked beam on elastic foundation subjected to moving mass can be derived as

$$\left(\left[M_{0}\right]+\left[M_{p}\right]\right)\{\ddot{q}\}+\left(\left[C_{0}(\{q\})\right]+\left[C_{p}\right]\right)\{\dot{q}\} +\left(\left[K_{bf}(\{q\})\right]+\left[K_{p}\right]\right)\{q\}=\{P\}, \quad (36)$$

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where $[M_0]+[M_p]=\sum_e [M^e]+\sum_{e_m}[M_p^e]$,

$$[C_0(\{q\})]+[C_p]=\alpha_R[M_0]+\beta_R[K_{bf}(\{q\})]+\sum_{e_m}[C_p^e],$$

$[K_{bf}]=\sum_e [K_{bf}^e]$, e is the number of normal elements, e_m is the number of elements directly subjected to moving mass and α_R, β_R are the Rayleigh damping coefficients.

This is a nonlinear differential equation system with time dependence coefficient that can be solved by using Newmark's direct integration and Newton-Raphson iteration method. A ANSYS program called CBF_Moving_Mass_2017 was conducted to solve equation (36). The code of the calculation program is written in the ANSYS 13.0 environment.

III. NUMERICAL ANALYSIS

Beam's length L = 8m, rectangular cross section with width b = 0.1m, height h = 0.2m; one end is in pinned connection, and the other end is in roller connection. Beam material with elastic modulus $E = 2.1\times10^{11}$ N/m^2, Poisson's ratio $\nu = 0.3$; density $\rho = 7800$ kg/m^3 is used. There is a V-shaped open crack in the center of the beam, and the crack's depth d = 0.1 m. Foundation stiffness $k_0 = 2\times10^4$ N/m^3. The used load is the material point with the mass m = 1000kg, moving along the beam with the velocity v = 36 km/h.

With the established program established, we calculate for 02 cases: Beam with crack (basic problem - BP) and without crack (comparison problem - CP) to see more clearly the impact on the dynamic response of the system when the cracks appear. The response results of deflection y, acceleration $\ddot{y}$, cutting force Q_y and normal stress σ_x at the midpoint of the beam (point A (4,0,0)) are shown in Table 2 and Figures 4, 5, 6, 7. Through this results, we realize that with cracked beam the whole displacement, acceleration of vertical displacements and normal stresses are greater than the beam without crack. This problem showed the dangers of crack to stiffness, stability of cracked beam on elastic foundation under moving loads.

Table 1. Summary of maximum values of calculated quantities

Quantities	y_{max} [cm]	$\ddot{y}_{max}$ [m/s2]	Q_y^{max} [N]	σ_x^{max} [N/m2]
BP (with crack)	0.373	20.994	109.889	3.316×10^7
CP (without crack)	0.191	0.125	119.421	1.079×10^7
Differents	1.95 times	167.95 times	0.92 times	3.07 times

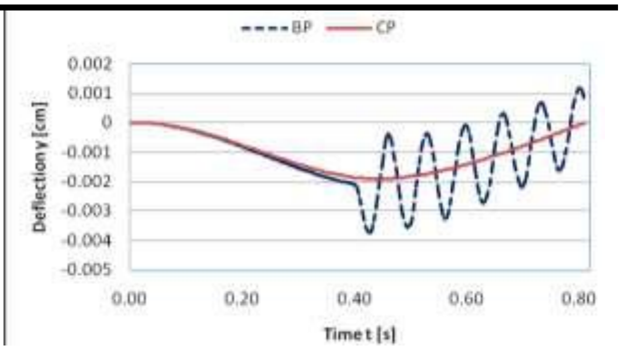

Fig. 4. Response of deflection y over time at the center cross section of the beam

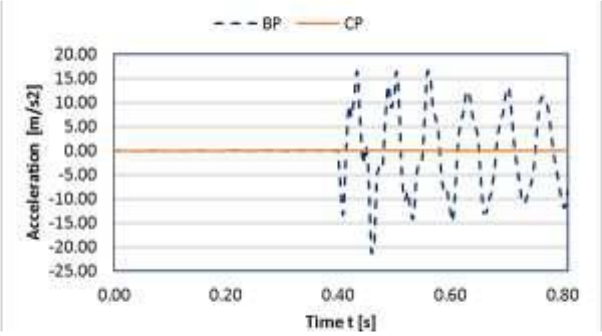

Fig. 5. Response of acceleration $\ddot{y}$ over time at the center cross section of the beam

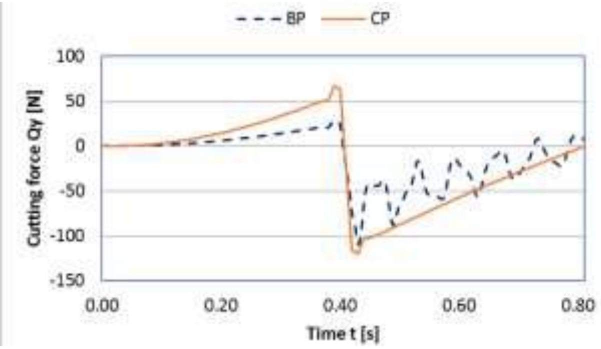

Fig. 6. Response of cutting force Q_y at the center cross section of the beam

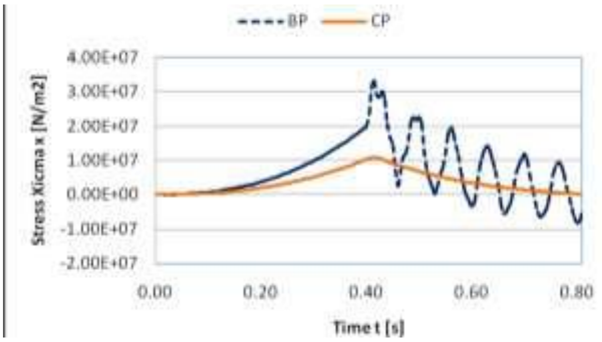

Fig. 7. Response of normal stress σ_x at the center cross section of the beam

The results show that the effect of cracks on the dynamic response of the beam is significant. For cracked beams,

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vibration of the beams increases after the mass moves through the crack.

3.1. Effect of elastic foundation stiffness

Studying the changes in maximum values of the displacement, internal force and direct stress of the beam under the elastic foundation stiffness, through the stiffness k0 of the spring ranging from 1×104N/m3 to 6×104N/m3. The results of changes in maximum values of the displacement, internal force and direct stress at the center cross section of the beam are shown in Table 3 and graphs in Figures 8 and 9.

Table 3. Summary of maximum values of quantities calculated based on k_0

k_0 [N/m]	1×10^4	2×10^4	4×10^4	6×10^4
y_{max} [cm]	0.459	0.373	0.279	0.228
$\ddot{y}_{max}$ [m/s^2]	17.963	20.994	21.762	21.480
Q_y^{max} [N]	113.220	109.889	104.525	99.085
σ_x^{max} [N/m^2] ($\times10^7$)	3.704	3.316	2.859	2.590

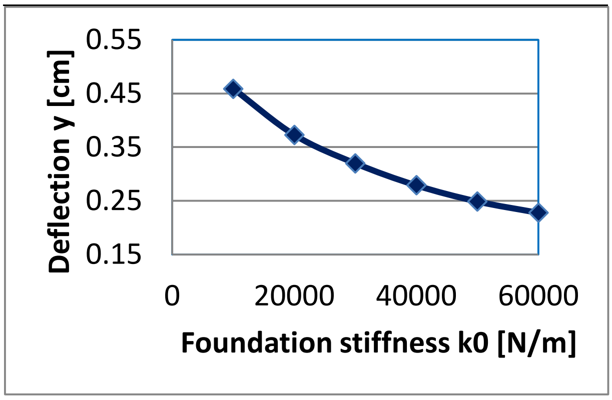

Fig. 8. y_{max} - k_0 relation

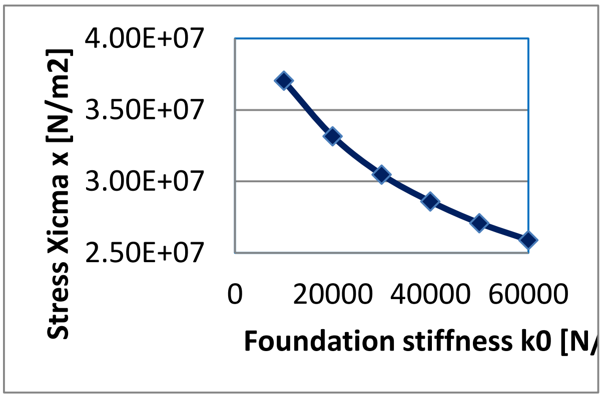

Fig. 9. σ_x^{max} - k_0 relation

It is observed that when the foundation stiffness increases, the maximum values of displacement and flexural moment decrease due to the increase in the system's overall stiffness. The maximum values of displacement and flexural moment decrease sharply when k_0 varies from 1×10^4N/m^3 to 3×10^4N/m^3, then the decreasing rate shall be slower.

3.2. Effect of load speed

Surveying the problem with a load speed v changes from 10m/s (36km/h) to 35m/s (126km/h). The results of the variations of the maximum values of deflection, acceleration, cutting force and stress at the midpoint of the beam based on v are shown in Table 4 and graphs in Figures 10 and 11.

Table 4. Summary of maximum values of quantities calculated based on v

v [km/h]	36	90	126
y_{max} [cm]	0.373	0.290	0.282
$\ddot{y}_{max}$ [m/s^2]	20.994	23.102	27.655
Q_y^{max} [N]	109.889	90.804	82.960
σ_x^{max} [N/m^2] ($\times10^7$)	3.316	2.890	2.797

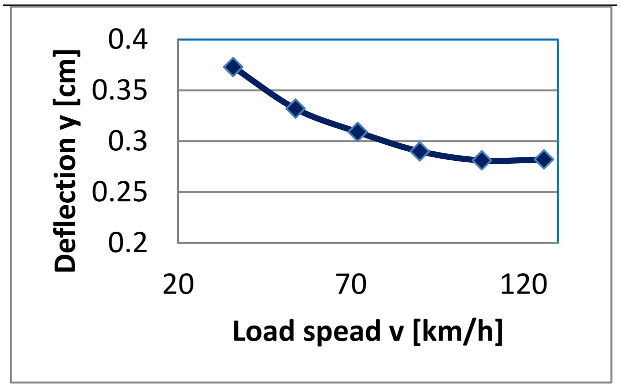

Fig. 10. y_{max} - v relation

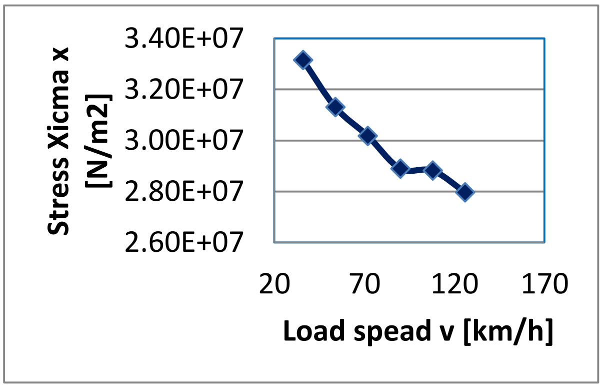

Fig. 11. σ_x^{max} - v relation

It is clear that when the moving speed of the load increases, the maximum values of displacement, internal force and stress in the beam decrease, when the moving speed of the load varies from 90 m/s to 110 m/s, the direct stress does not change much, then decreases sharply.

3.3. Effect of crack location

This example studies the changes in maximum values of displacement and internal force of the beam according to the crack location, giving the cracks located far from the

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beam's ends the distances L/4, L/2, 3L/4. Results of extreme values of the responses at the calculated points are shown in Tab. 5 and graphs in Figures 12, 13, 14, 15. When mass moving through the crack, the beam vibrates amplitude and frequency, which shows that the stabilization of the beam oscillations during this period decreased.

Table 5. Summary of maximum values of quantities calculated according to the crack location

Crack location (from left end)	L/4	L/2	3L/4
y_{max} [m]	0.257	0.373	0.279
$\ddot{y}_{max}$ [m/s^2]	16.347	20.994	19.855
Q_y^{max} [N]	75.374	109.889	63.418
σ_x^{max} [N/m^2] ($\times 10^7$)	1.453	3.316	1.133

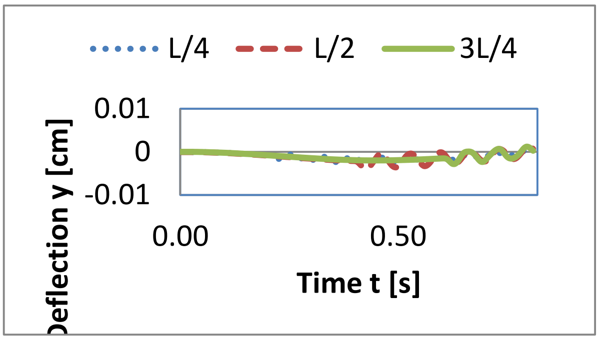

Fig. 12. Response of y according to the crack location

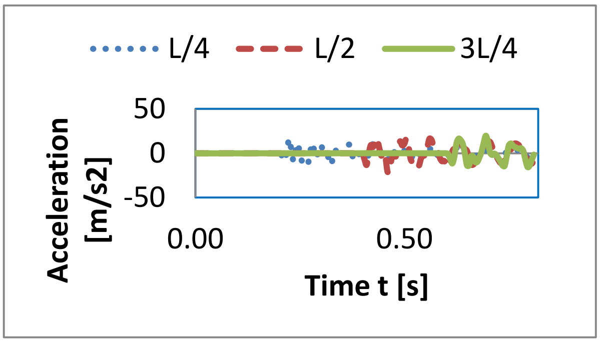

Fig. 13. Response of $\ddot{y}$ according to the crack location

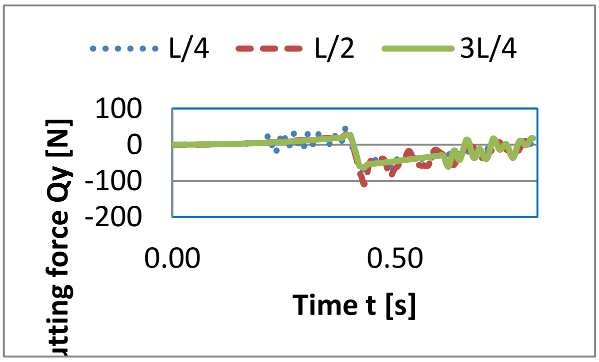

Fig. 14. Response of Q_y according to the crack location

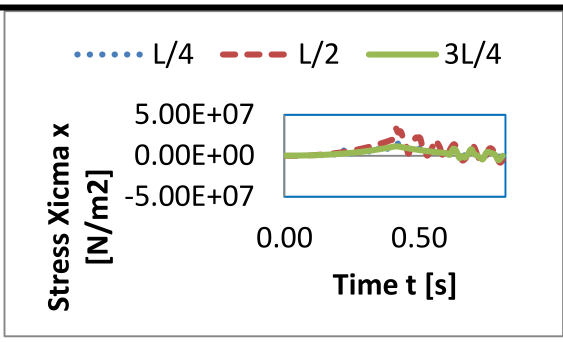

Fig. 15. Response of σ_x according to the crack location

Crack location changes making the maximum responses of displacement, stress and internal force in the beams change significantly; when the crack is in the center of the beam, the above quantities reach the maximum values, so, the beam is most dangerous when there is a crack appearing in this position.

IV. CONCLUSION

A conclusion section must be included and should indicate clearly the advantages, limitations, and possible applications of the paper. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

The nonlinear dynamics analysis of cracked beams resting on a Winkler foundation subjected to a moving mass using the finite element method has been presented. A two-node beam element based on Euler-Bernoulli beam theory, taking the effect of crack and foundation support, was derived and employed in the analysis. The dynamics response of the beams, including the time histories for deflection, acceleration and normal stress, was computed with the aid of Newmark method. The effect of loading parameters, foundation stiffness and crack location on the dynamic response of the beams has been examined and highlighted. The main conclusions can be summarized as follows:

The beam element and computer code developed in the present work are accurate in evaluating the dynamic characteristics of cracked beams subjected to moving masses.

The elastic foundation plays an important role in the dynamic response of the cracked beams under a moving mass. Both the dynamic deflection and normal stress are significaly decreased by the increase of the foundation stiffness.

The dynamic response of the cracked beams, as in case of the uncracked beams, is governed by the moving mass speed. With the moving speeds in the range considered in this paper, both the dynamic deflection and normal stress decreased when increasing the moving speed.

The maximum dynamic deflection and normal stress are significantly influenced by the crack location. The deflection and normal tress attain the largest values when the crack is located at the midpoint of the beam. Thus, from an engineering point of view, the midpoint crack is the most dangerous one.

The results obtained in this paper help to select appropriate parameters, the solution for structural reinforcement cracked beam on elastic foundation under moving load and applications in transportation techniques such as the train rails.

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